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STAT 412, HW 7
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Ex. 10.27/p. 435

Herbicide A	}	66	67	74	73	75	64	$S_1^2 = 22.17$	$T_1 = 419$	$\bar{y}_1 = 69.83$
Herbicide B		85	84	76	82	79	86	$S_2^2 = 14.8$	$T_2 = 492$	$\bar{y}_2 = 82$
Herbicide C		91	93	88	87	90	86	$S_3^2 = 6.97$	$T_3 = 535$	$\bar{y}_3 = 89.17$

 $G = 1446$ HARTLEY'S F_{max} TEST

$$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \quad \checkmark$$

 H_a : Not all variances are equal. \checkmark

$$T.S.: F_{max} = \frac{S^2_{max}}{S^2_{min}} = \frac{22.17}{6.97} = 3.18 \quad \checkmark$$

R.R: Reject H_0 if $F_{max} > F_{max}(\alpha, t, r_i - 1)$

$$F_{max}(.05, 3, 5) = 10.8 \quad \checkmark$$

C: Since 3.18 is not greater than 10.8,

we assume that the variances are not significantly different. (Don't reject H_0)So we perform ANOVA. \checkmark

ANOVA

$$H_0: \mu_1 = \mu_2 = \mu_3$$

H_a : At least one of μ 's differs from the rest.

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$$SS_T = \sum y_{ij}^2 - \frac{G^2}{n} = 117,528 - \frac{1446^2}{18} = 1366$$

$$SS_B = \frac{\sum T_i^2}{n_i} - \frac{G^2}{n} = \frac{703,850}{6} - \frac{1446^2}{18} = 1146.33$$

$$SS_W = SS_T - SS_B = 1366 - 1146.33 = 219.67$$

$$MS_B = \frac{SS_B}{t-1} = \frac{1146.33}{2} = 573.165$$

$$MS_W = \frac{SS_W}{n-t} = \frac{219.67}{15} = 14.64$$

$$F = \frac{MS_B}{MS_W} = \frac{573.165}{14.64} = 39.15$$

<u>Source</u>	<u>SS</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Between	$SS_B = 1146.33$ ✓	$t-1 = 2$	$MS_B = \frac{SS_B}{t-1} = 573.165$	$F = \frac{MS_B}{MS_W} = 39.15$ ✓
Within	$SS_W = 219.67$ ✓	$n-t = 15$	$MS_W = \frac{SS_W}{n-t} = 14.64$	
Total	$SS_T = 1366$ ✓	$n-1 = 17$		

R.R: Reject H_0 if $F > F_{(\alpha, t-1, n-t)}$

$$F_{(0.05, 2, 15)} = 3.68$$

C: Since 39.15 is greater than 3.68, we reject H_0 , which means at least one of μ 's is different from the rest.

Now, we can perform LSD.

$$\begin{aligned}
LSD &= t_{(\alpha/2, n-t)} \cdot \sqrt{S_w^2 \left(\frac{1}{n_i} + \frac{1}{n_j} \right)} \\
&= t_{(0.025, 15)} \cdot \sqrt{(14.64) \left(\frac{1}{6} + \frac{1}{6} \right)} \\
&= 2.131 \sqrt{(14.64) \left(\frac{1}{3} \right)} = \\
&= 4.71 \quad \checkmark
\end{aligned}$$

$|69.83 - 82| = 12.17$

$|69.83 - 89.17| = 19.34$

$|82 - 89.17| = 7.17$

- $H_0: \mu_1 = \mu_2$
 $H_a: \mu_1 \neq \mu_2$ } since $12.17 > LSD = 4.71$, Reject H_0 ✓

- $H_0: \mu_1 = \mu_3$
 $H_a: \mu_1 \neq \mu_3$ } since $19.34 > LSD = 4.71$, Reject H_0 ✓

- $H_0: \mu_2 = \mu_3$
 $H_a: \mu_2 \neq \mu_3$ } since $7.17 > LSD = 4.71$, Reject H_0 ✓

We conclude that all means are different from each other. ✓

good

Ex. 10.32 / p. 436

Machine A	105	3	90	217	22	$S_1^2 = 7128.3$	$T_1 = 437$	$\bar{y}_1 = 87.4$
Machine B	56	43	1	37	14	$S_2^2 = 497.7$	$T_2 = 151$	$\bar{y}_2 = 30.2$
Machine C	183	144	219	86	39	$S_3^2 = 5263.7$	$T_3 = 671$	$\bar{y}_3 = 134.2$
							$G = 1259$	$\bar{y} = 83.93$

HARTLEY'S F_{\max} TEST.

$$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \quad \checkmark$$

H_a : Not all variances are equal. \checkmark

$$T.S: F_{\max} = \frac{S^2_{\max}}{S^2_{\min}} = \frac{7128.3}{497.7} = 14.32 \quad \checkmark$$

RR: Reject H_0 if $F_{\max} > F_{\max}(\alpha, t, n_i - 1)$

$$F_{\max}(0.05, 3, 4) = 15.5 \quad \checkmark$$

c: Since 14.32 is not greater than 15.5,

we assume that the variances are not significantly different. (Don't reject H_0)

So we perform ANOVA. \checkmark

ANOVA.

$H_0: \mu_1 = \mu_2 = \mu_3$ ✓

$H_a:$ At least one of μ 's differs from the rest. ✓

$$SS_T = \sum y_{ij}^2 - \frac{G^2}{n} = 184,361 - \frac{1259^2}{15} = 78688.93$$

$$SS_B = \sum \frac{T_i^2}{n_i} - \frac{G^2}{n} = \frac{664011}{5} - \frac{1259^2}{15} = 27130.13$$

$$SS_W = SS_T - SS_B = 78688.93 - 27130.13 = 51558.8$$

$$MS_B = \frac{SS_B}{t-1} = \frac{27130.13}{2} = 13565.07$$

$$MS_W = \frac{SS_W}{n-t} = \frac{51558.8}{12} = 4296.57$$

$$F = \frac{MS_B}{MS_W} = \frac{13565.07}{4296.57} = 3.16$$

Source	SS	df	MS	F
Between	$SS_B = 27130.13$ ✓	$t-1 = 2$	$MS_B = \frac{SS_B}{t-1} = 13565.07$	$F = \frac{MS_B}{MS_W} = 3.16$ ✓
Within	$SS_W = 51558.8$ ✓	$n-t = 12$	$MS_W = \frac{SS_W}{n-t} = 4296.57$	
Total	$SS_T = 78688.93$ ✓	$n-1 = 14$		

R.R. Reject H_0 if $F > F(\alpha, t-1, n-t)$

$$F(.05, 2, 12) = 3.89$$
 ✓

C: Since 3.16 is not greater than 3.89,

We don't reject H_0 . There is not enough evidence to reject that the three means are equal. Therefore, we don't need to perform LSD.