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STAT 512
C.T. Gaskins
Problems

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1. An agronomist has developed a new variety of peas which he believes has better yield than the variety that is currently being used. Call these varieties "NEW" and "OLD".

   a. Set up the null and alternative hypothesis.

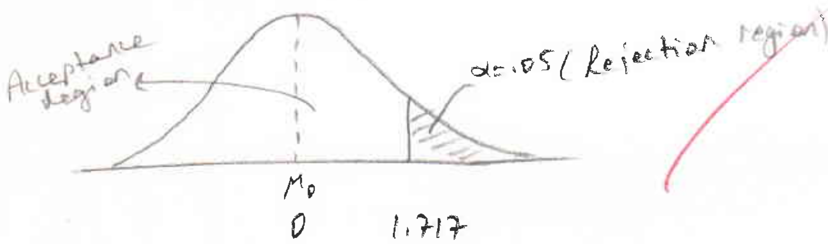
   $H_0: \mu_n = \mu_o \quad (\mu_n - \mu_o = 0)$

   $H_a: \mu_n > \mu_o \quad (\mu_n - \mu_o > 0)$

   b. Twelve independent yield (kg/hectare) are obtained for each variety. The agronomist wants the chance of his being wrong if he rejects the true null hypothesis to be 5 percent.

      1. Draw a diagram showing the acceptance and rejection regions. You must give numerical value(s) to define these regions.

      $\alpha = .05 \qquad df = 12+12-2 = 22$



$t_{.05,22} = 1.717$

      2. What test statistic should be used deciding whether to accept or reject the null hypothesis.

      $$t_{test} = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{12}+\frac{1}{12}}} = \frac{\bar{y}_1 - \bar{y}_2}{S_p/\sqrt{6}}$$

      3. Tell what values of the test statistic will result in rejection of the null hypothesis

      $t_{.05,22} = 1.717$ Reject H_0 if $t_{test} > 1.717$

2. Another agronomist imported 2 varieties developed in another country. He wants to set up a test to determine if one variety is better than the other. In testing the null hypothesis he wants to be 99 percent certain that he will accept the null if it is true. Eight samples for each variety were measured.

   a. Answer all questions given in problem 1.

   2 tailed test.

   $\alpha = .01$

   $df = 8+8-2 = 14$

   $H_0: \mu_1 = \mu_2 \ (\mu_1 - \mu_2 = 0)$

   $H_a: \mu_1 \neq \mu_2 \ (\mu_1 - \mu_2 \neq 0)$

   $t_{.005,14} = 2.977$

   $$t_{test} = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{8}+\frac{1}{8}}} = \frac{\bar{y}_1 - \bar{y}_2}{S_p/2}$$

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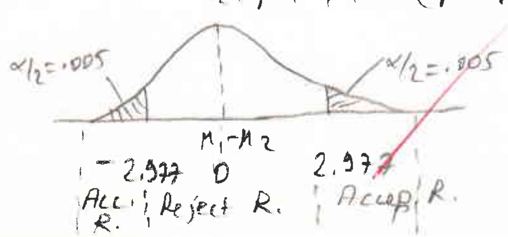

Reject H_0 if $|t| > 2.977$

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Problems - β Error Rates, Power of the Test

A statistician conducts a test in which the null hypothesis is $\mu_o = 160$ and the alternative is $\mu_A = 180$. He sets the Type 1 error rate at 5%. The parametric standard error of the mean is 7. Diagram this problem and compute the Type II error rate. What is the power of this test?

$H_o: \mu_o = 160$
$H_a: \mu_a = 180$
$\alpha = .05$
$S = 7.0$

$Y = (1.645 \times 7) + 160 = 171.5$

$z = \frac{180 - 171.5}{7} = 1.214$

$\beta = P[z < -1.214] = .11237$

Power $= 1 - \beta = .88763 = \boxed{88.763\ \%}$

2. An experiment was conducted comparing 2 treatments (groups) H_o was $\mu_1 - \mu_2 = 0$ and H_A was $\mu_1 - \mu_2 = .5$. If $\alpha = .01$, $\sigma_{\bar{Y}_1 - \bar{Y}_2} = .2$.
What is the Type II error rate?

$H_o: \mu_1 - \mu_2 = 0$
$H_a: \mu_1 - \mu_2 = .5$
$\alpha = .01$
$\sigma_{\bar{y}_1 - \bar{y}_2} = .2$

$\beta = P\left[z < z_{.01} - \frac{|\mu_1 - \mu_2|}{\sigma_{\bar{y}_1 - \bar{y}_2}}\right] = P\left[z < 2.33 - \frac{.5}{.2}\right] = P[z < -.17]$

$\boxed{\beta = .43251}$

3. In a paired test the null hypothesis is $H_o: \mu_D = 22$ and the alternative $\mu_A = 19.5$. σ_D is known to be 2.0 and the number of pairs is 16. Compute the power of the test when $\alpha = .05$.

$S_{\bar{D}} = 2/\sqrt{16}$
$S_{\bar{D}} = .5$

$\beta = P\left[z < 1.753 - \frac{2.5}{.5}\right] = P[z < -3.247] = .00058$

Power $= 1 - \beta = 1 - .00058 = .99942 = 99.942\ \%$

4. All other factors being equal: (increase or decrease)

a. Increasing α causes a <u>decrease</u> in β.

b. Decreasing α causes a <u>decrease</u> in the power of the test.

c. Increasing n causes a <u>decrease</u> in β.

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