

STATISTICS 512
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Problem - Analysis of Covariance

Yield (Y) in bushels per acre and the percents of stem canker infection (X) in a randomized block experiment comparing four lines of soybeans.

Block	LINES								TOTALS	
	A		B		C		D		X	Y
	X	Y	X	Y	X	Y	X	Y	X	Y
1	19.3	21.3	10.1	28.3	4.3	26.7	14.0	25.1	47.7	101.4
2	29.2	19.7	34.7	20.7	48.2	14.7	30.2	20.1	142.3	75.2
3	1.0	28.7	14.0	26.0	6.3	29.0	7.2	24.9	28.5	108.6
4	6.4	27.3	5.6	34.1	6.7	29.0	8.9	29.8	27.6	120.2

SUMMARY DATA										
ΣX	55.9		64.4		65.5		60.3		246.1	
ΣY		97.0		109.1		99.4		99.9		405.4
ΣX^2	1267.09		1533.46		2426.31		1239.09		6465.95	
ΣY^2		2410.76		3068.19		2610.98		2542.07		10632
SSX	485.8875		496.62		1353.7475		330.0675			
SSY		58.51		92.4875		140.89		47.0675		
ΣXY	1189.75		1559.08		1200.35		1402.92		5352.1	
SCP	-165.825		-197.43		-427.325		-103.0725			

SS = Sums of Squares; SCP = Sums of Cross products

1. Do analysis of Variance on Y(F=.96425)
2. Plot X(Horizontal) versus Y(Vertical) indicating the treatment.
3. Do analysis of covariance (F=4.72)
4. Compute adjusted line means
5. Compare line adjusted means using Duncans New Multiple Range Test.

Analysis of covariance

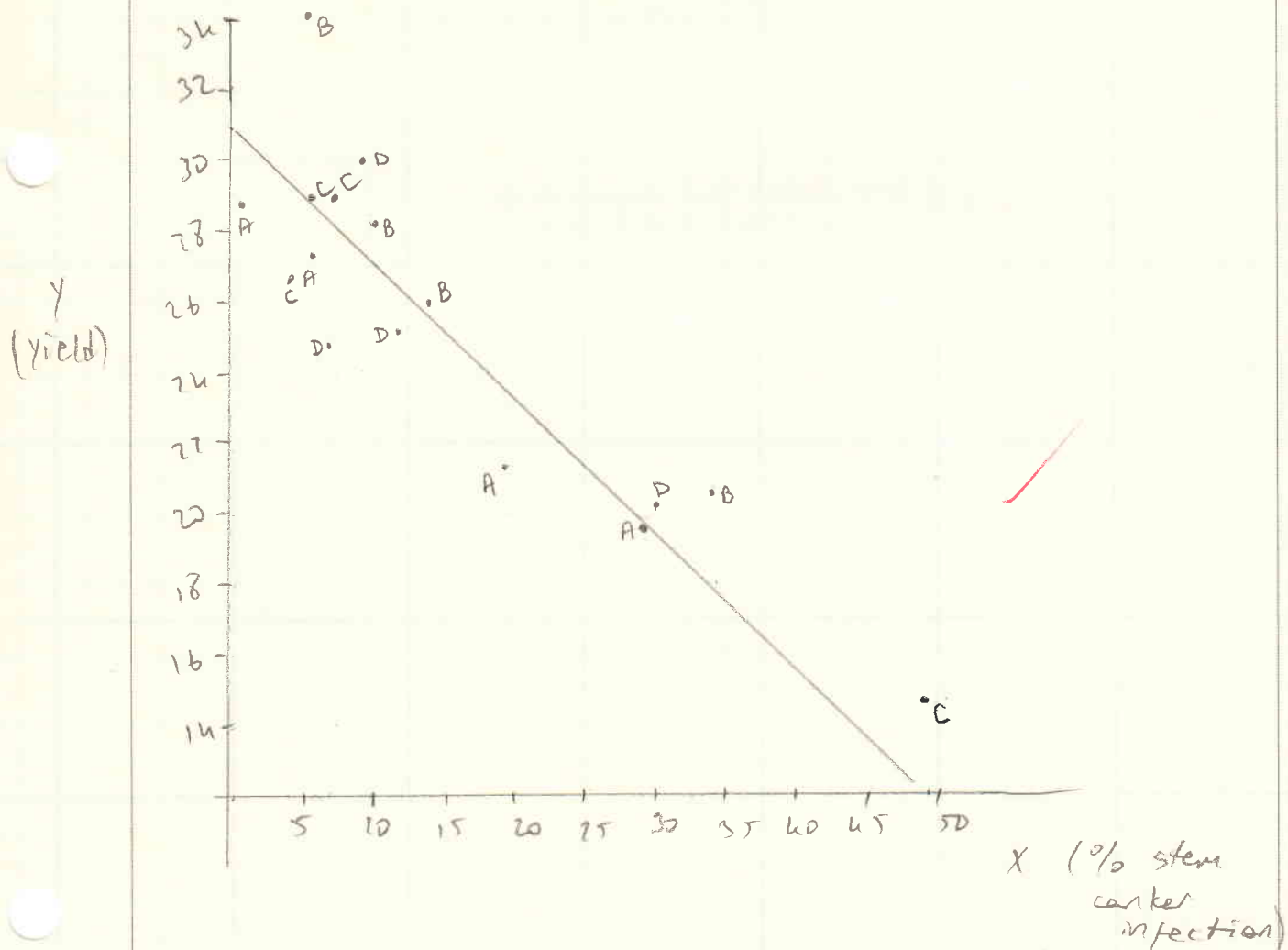
Source	df	SS	MS	F	F _{critical}
Total	16-1=15	360.18			
Block	4-1=3	272.93	90.98	12.4 *	3.86
Variety	4-1=3	21.23	7.07	.96	3.86
Error	3x3=9	66.03	7.34		

$$SS(\text{total}) = 10632 - (405.4)^2 / 16 = 360.1775$$

$$SS(\text{block}) = 42209.1 / 4 - 10271.8 = 272.9275$$

$$SS(\text{Variety}) = 41172.2 / 4 - 10271.8 = 21.2225$$

$$SS(\text{Error}) = 360.18 - 272.93 - 21.22 = 66.03$$



3)

Source	df	SS			df	SS	MS	F
		XX	XY	YY				
Total	15	2680.62	-883.5	360.2				
Block	3	2239.32	-747.9	272.9				
Variety (treat)	3	14.30	10.2	21.2				
Error	9	427.0	-145.7	66.0	8	16.33 (E ₁)	2.04	
Var. + Error	12	441.3	-135.5	87.25	11	45.65 (E ₀)	4.15	
Var. Adj.					3	29.32	9.77 * 4.79	

$$SS_{XX} = 6465.95 - (246.1)^2 / 16 = 2680.62$$

$$SS(\text{Block}) = 24098.6 / 4 - 3785.33 = 2239.32$$

$$SS(\text{Var.}) = 3799.62 - 3785.33 = 14.3018$$

$$SS(\text{Error}) = SS(\text{Tot}) - SS(\text{Block}) - SS(\text{Treat}) = 427.00$$

$$(\text{Tot}) SS_{XY} = 5352.1 - 6235.558 = -883.46$$

$$(\text{block}) SS_{XY} = 21950.36 / 4 - 6235.558 = -747.968$$

$$(\text{Var}) SS_{XY} = 24983.01 / 4 - 6235.558 = 10.19$$

$$\text{Error} = \text{Total} - \text{Block} - \text{Var.} = -145.683$$

$$E_1 \text{ Error } YY - \frac{(E_{XY})^2}{E_{XX}} = 66.0 - \frac{(-145.7)^2}{427} = 16.33$$

$$E_0 \text{ Adj. Var. + Error } SS = S_{YY} - \frac{(S_{XY})^2}{S_{XX}} = 87.25 - \frac{(-135.5)^2}{441.3} = 45.65$$

$$(E_0 - E_1) \text{ Adj. Var. (Treat) } SS = \text{Adj. Var. + Error} - \text{Adj. Error}$$

$$E_0 - E_1 = 45.65 - 16.33 = 29.32$$

4)

Adj. Line Means

\bar{y}_i	\bar{x}_i	\bar{x}_{11}	$\bar{x}_i - \bar{x}_{11}$
24.25	13.98	15.38	-1.406
27.28	16.1	15.38	0.719
24.85	16.38	15.38	0.994
24.98	15.08	15.38	-0.306

$$\hat{y} = \bar{y} - b_{Y,X}(\bar{x}_i - \bar{x}_{11})$$

$24.25 - (-.389 \times -1.406) = 23.79$
 $27.28 - (-.329 \times 0.719) = 27.52$
 $24.85 - (-.329 \times 0.994) = 25.18$
 $24.98 - (-.329 \times -0.306) = 24.88$

5)

A	D	C	B
23.79	24.88	25.18	27.52

P	2	3	4
9.08(p,8)	3.26	3.29	3.47
LSR:	3.30	3.43	3.54

$$S_{\hat{y}_B - \hat{y}_A} = \sqrt{2.04 \left[\frac{2}{4} + \frac{(16.1 - 13.98)^2}{427} \right]} = 1.02$$

$$S_{\hat{y}_B - \hat{y}_D} = \sqrt{2.04 \left[\frac{2}{4} + \frac{(16.1 - 15.08)^2}{427} \right]} = 1.012$$

$$S_{\hat{y}_C - \hat{y}_A} = \sqrt{2.04 \left[\frac{2}{4} + \frac{(16.38 - 13.98)^2}{427} \right]} = 1.025$$

$$S_{\hat{y}_D - \hat{y}_A} = \sqrt{2.04 \left[\frac{2}{4} + \frac{(15.08 - 13.98)^2}{427} \right]} = 1.013$$

Comparison

B-A	3.73	> 3.54	* (S)
B-D	2.64	< 3.43	NS
C-A	1.39	< 3.46	NS
D-A	1.09	< 3.30	NS

A	D	C	B
23.79	24.88	25.18	27.52

Treatment A and B are significantly different.