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Statistics 512
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Problem - Analysis of Covariance - Completely Random Design

An experiment was conducted to compare the effects of 6 diets on weight gain (Y) of rats. How these gains were affected by the continuous independent variable calorie intake (X) was also considered. The data and summary values are given below.

FOOD INTAKE, X (10-CALORIE UNITS), AND GAIN IN WEIGHT, Y (GRAMS), OF 60 RATS RECEIVING SIX RATIONS IN LOTS OF 10

	Lot 1		Lot 2		Lot 3		Lot 4		Lot 5		Lot 6	
Rat	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
1	108	73	99	98	194	94	165	90	124	107	140	49
2	136	102	117	74	198	79	164	76	95	95	177	82
3	138	118	90	56	196	96	161	90	116	97	189	73
4	159	104	141	111	198	98	159	64	112	80	142	86
5	146	81	106	95	210	102	175	86	123	98	216	81
6	141	107	112	88	196	102	135	51	110	74	200	97
7	175	100	110	82	230	108	132	72	137	74	255	106
8	149	87	117	77	222	91	190	90	105	67	173	70
9	174	117	111	86	220	120	145	95	135	89	153	61
10	176	111	122	92	228	105	142	78	126	58	160	82
Sums	1,502	1,000	1.125	859	2.092	995	1,568	792	1.183	839	1.805	787
SX^2 / SY^2	229,760	102,062	128,245	75.819	439.544	100,075	248,886	64,462	141.525	72,613	337,433	64,401
SXY	151,846		97,776		208,892		125,370		99,195		145,872	

Experiment totals: $SX = 9{,}275$ $SY = 5{,}272$ $SXY = 828{,}951$
$SX^2 = 1{,}525{,}393$ $SY^2 = 479{,}432$

1. Plot the data. Identify data points by treatment

2. Do analysis of variance and analysis of covariance.

3. Make a table of the unadjusted and adjusted diet means.

4. Compute the regression coefficients separately for each treatment.

5. Compute and plot (on the plot for part 1) the within diet regression lines (Daily gain on weight)

6. Test H_0: $\beta_1 = \beta_6$, that the regression coefficients for the diets 1 and 6 are equal.

PROBANOC.512

CRD - Analysis of Covariance

2) Analysis of Variance

Source	df	SS	MS	F	Fcri
Total	59	16198.93	-		
Diets	5	4612.93	922.59	4.31 *	2.37
Error	54	11564	214.15		

SS(total) = 479432 − 463233.07 = 16198.93

SS(diet) = $1000^2 + \dots + 787^2/10 - 463233.07 = 4612.93$

SS(error) = 11564

Covariance (Fcri = 2.37)

Source	df	SS XX	SS XY	SS YY	df	SS	MS	F
total	59	91632.6	13987.7	16198.9				
diets	5	67662.7	5521.0	4612.9				
Error	54	23969.9	8466.7	11586	53	$E_1 = 8595.4$	162.18	
Adj. (treat+Error)					48	$E_0 = 14063.7$	293.0	
Adj. (Error)					5	5468.3 $(E_0 - E_1)$	1093.7 *	6.74

Total $S_{xx} = 91632.6$

Diets $T_{xx} = 1505^2 + \dots + 1805^2/10 - 1433760.4 = 67662.7$

Error $E_{xx} = 91632.6 - 67662.7 = 23969.9$

total $S_{xy} = 828.951 - 9275 \times 5272/60 = 13987.7$

Diets $T_{xy} = 1502 \times 1000 + \dots + 1805 \times 787/10 - 814963.3 = 5521$

Error $E_{xy} = 13987.7 - 5521 = 8466.7$

Y adjusted for x

$$SS_{regression} = b_{y.x} = \frac{(Exy)^2}{Exx} = \frac{8466.7^2}{23969.9} = 2990.6$$

$$\text{Adj. SS error} = E_1 = E_{yy} - SS_{Reg} = 11586 - 2990.6 = 8595.4$$

$$\text{Adj. SS (Treat. + Error)} = E_0 = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = 16198.9 - \frac{13987.7^2}{91632.6}$$

$$= 14063.7$$

$$\text{Adj. (Error)} = E_0 - E_1 = 5468.3$$

3) Table of unadj. and adj. diet means.

$\bar{x} = 154.58$ $\quad b_{y.x} = \frac{Exy}{Exx} = \frac{8466.7}{23969.9} = .35$

(Diets) Treat.	$\bar{X}_{i.}$	$\bar{Y}_{i.}$	$\bar{X}_{i.} - \bar{X}_{..}$	$b_{y.x}(\bar{X}_{i.} - \bar{X}_{..})$	$\hat{\bar{Y}}_{i.} = \bar{Y}_{i.} - b_{y.x}(\bar{X}_{i.} - \bar{X}_{..})$
1	150.2	100	-4.38	-1.533	101.53
2	112.5	85.9	-42.08	-14.73	100.63
3	209.2	99.5	54.62	19.12	80.38
4	156.8	79.2	2.22	.78	78.42
5	118.3	83.9	-36.28	-12.70	96.60
6	180.5	78.7	25.92	9.07	69.63

YY

5) Regression coefficients for each treatment

Treat. 1/

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 229760 - \frac{1502^2}{10} = 4159.6$

$S_{xy} = \Sigma xy - \frac{(\Sigma x \Sigma y)}{n} = 151846 - (1502)(1000)/10 = 1646$

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{1646}{4159.6} = .4$

$\hat{\beta}_0 = \bar{Y}_i - \hat{\beta}_1(\bar{x}) = 100 - .4(150.2)$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1(x) = 39.92 + .4x$ $\hat{\beta}_0 = 39.92$

Tr. 2/

$S_{xx} = 128245 - 1125^2/10 = 1682.5$

$S_{xy} = 97776 - (1125)(859)/10 = 1138.5$

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{1138.5}{1682.5} = .7$ $\hat{\beta}_0 = 85.9 - .7(112.5) = 7.15$

$\hat{y} = 7.15 + .7x$

Tr. 3/

$S_{xx} = 439544 - 2092^2/10 = 1897.6$

$S_{xy} = 208892 - (2092)(995)/10 = 738$

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = 738/1897.6 = .4$

$\hat{\beta}_0 = 99.5 - .4(209.2) = 15.82$

$\hat{y} = 15.82 + (.4)x$

TT. 4/

$S_{xx} = 248886 - 1568^2/10 = 3023.6$

$S_{xy} = 125370 - (1568)(792)/10 = 1184.4$

$\hat{\beta}_1 = 1184.4/3023.6 = .4$

$\hat{\beta}_0 = 79.2 - .4(156.8) = 16.48$

$\hat{y} = 16.48 + .4X$

TT. 5/

$S_{xx} = 141525 - 1183^2/10 = 1576.1$

$S_{xy} = 99195 - (1183)(839)/10 = -58.7$

$\hat{\beta}_1 = -58.7/1576.1 = -.04$

$\hat{\beta}_0 = 83.9 + .04(118.3) = 88.63$

$\hat{y} = 88.63 - .04X$

TT. 6/

$S_{xx} = 337433 - 1805^2/10 = 11630.5$

$S_{xy} = 145872 - (1805)(787)/10 = 3818.5$

$\hat{\beta}_1 = 3818.5/11630.5 = .33$

$\hat{\beta}_0 = 78.7 - .33(180.5) = 19.14$

$\hat{y} = 19.14 + .33X$

6) $H_0: \beta_1 = \beta_6$

$$t = \frac{\beta_1 - \beta_6}{\sqrt{MSE\left(\frac{1}{\Sigma x_1^2} + \frac{1}{\Sigma x_6^2}\right)}} = \frac{.4 - .33}{\sqrt{162.18\left(\frac{1}{4159.6} + \frac{1}{11630.5}\right)}}$$

$$= \frac{.07}{.23} = .30$$

$t_{.05, 48} = 2$

Since $.30 < 2$, we accept Ho that regression coefficients for the diets 1 and 6 are equal.

4) Treatment

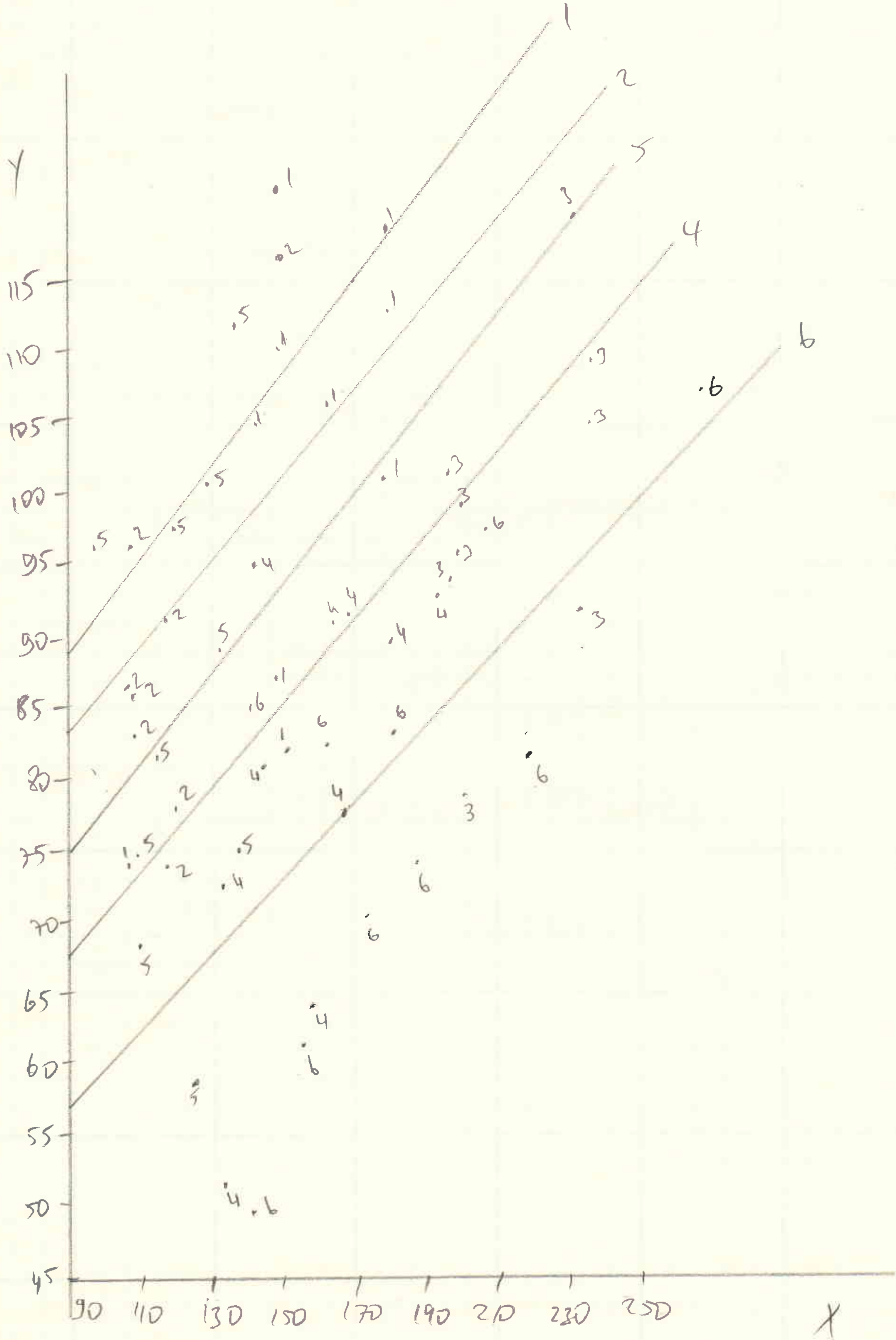
Treatment	
1	$\hat{Y}_1 = 39.92 + .4X$
2	$\hat{Y}_2 = 7.15 + .7X$
3	$\hat{Y}_3 = 15.82 + .4X$
4	$\hat{Y}_4 = 16.48 + .4X$
5	$\hat{Y}_5 = 88.63 + .04X$
6	$\hat{Y}_6 = 19.14 + .33X$

Computed $\hat{X}$ and $\hat{Y}$ values:

	Tr. 1		Tr. 2		Tr. 3	
	X	Y	X	Y	X	Y
1-	108	83.12	99	76.45	194	93.42
2-	136	96.32	117	89.05	198	95.02
3-	138	95.12	90	70.15	196	96.22
4-	159	103.52	141	105.85	198	95.02
5-	146	98.32	106	81.35	210	99.82
6-	141	96.32	112	85.55	196	94.22
7-	175	109.92	110	84.15	230	107.82
8-	149	99.52	117	89.05	222	104.62
9-	174	109.52	111	84.85	220	103.82
10-	176	110.24	122	92.55	228	107.02

Tr. 4		Tr. 5		Tr. 6	
X	Y	X	Y	X	Y
165	82.48	124	83.76	140	65.33
164	82.02	95	84.83	177	77.55
161	80.88	116	83.99	189	81.51
159	80.08	112	84.15	142	66.80
175	86.48	123	83.71	216	90.42
135	70.48	110	84.23	200	85.14
132	69.28	137	83.15	255	103.29
190	92.48	105	84.43	173	76.23
145	74.48	135	83.22	153	69.63
142	73.29	126	84.59	160	71.96

Q-1 Data points by treatments.



Q-5/ Plot within diet regression lines.