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STAT 512
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Problem

Oxygen consumption rates of two species of limpets, <u>Acmaea scabra</u> and <u>Acmaea digitalis</u>, was evaluated at at three concentrations of seawater. The variable measures is $\mu l\ O_2$/mg. dry body wt./min at 22° C. There are 8 replicates per combination of species and salinity.

FACTOR A: SPECIES

FACTOR B: SEAWATER CONCENTRATION

FACTOR B: SEAWATER CONCENTRATION	Acmaea scabra			Acmaea digitalis			
100%	7.16	8.26	$Y_{11.}=84.49$	6.14	6.14	$Y_{12.}=59.43$	$\bar{Y}_{1..}=8.995$
	6.78	14.00	$\bar{Y}_{11.}=10.56$	3.86	10.00	$\bar{Y}_{12.}=7.43$	
	13.60	16.10		10.40	11.60		
	8.93	9.66		5.49	5.80		
75%	5.20	13.20	$Y_{21.}=63.12$	4.47	4.95	$Y_{22.}=58.7$	$\bar{Y}_{2..}=7.61$
	5.20	8.39	$\bar{Y}_{21.}=7.89$	9.90	6.49	$\bar{Y}_{22.}=7.34$	
	7.18	10.40		5.75	5.44		
	6.37	7.18		11.80	9.90		
50%	11.11	10.50	$Y_{31.}=97.39$	9.63	14.50	$Y_{32.}=98.61$	$\bar{Y}_{3..}=12.25$
	9.74	14.60	$\bar{Y}_{31.}=12.17$	6.38	10.20	$\bar{Y}_{32.}=12.33$	
	18.80	11.10		13.40	17.70		
	9.74	11.80		14.50	12.30		
	$\bar{Y}_{.1.}=10.21$			$\bar{Y}_{.2.}=9.03$			

a = 2
b = 3
n = 8
N = 48

SUMS TABLE

	A 1	A 2	B SUMS
B 1	84.49	59.43	143.92
2	63.12	58.70	121.82
3	97.39	98.61	196.00
A SUMS	245.00	216.74	461.74

$$\sum_{ijk} Y^2_{ijk} = 5065.1530$$

1. Give the linear model describing an X value.
2. Calculate the class and subclass means.
3. Partition the sums of squares using the usual analysis of variance techniques.
4. Discuss the meaning of each of the F-ratios regarding significance and give the null and alternative hypothesis related to each F test. Indicate what other tests should be made.
5. Compute and <u>interpret</u> a t-test for the difference between the species means.
6. Partition the 2 degrees of freedom for salinity using orthogonal polynomials and make the appropriate tests of significance. <u>Interpret</u> the results of these tests.

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(1) $X_{ijk} = \mu + A_i + B_j + (AB)_{ij} + e_{ijk}$

A_i = the main effect for factor A

B_j = " " " " " B

$(AB)_{ij}$ = the interaction effect.

(2) CLASS MEANS

A	B
$\bar{X}_{\cdot 1} = 245/24 = 10.21$	$\bar{Y}_{1\cdot} = 143.92/16 = 8.995$
$\bar{Y}_{\cdot 2} = 216.74/24 = 9.03$	$\bar{Y}_{2\cdot} = 121.82/16 = 7.61$
	$\bar{Y}_{3\cdot} = 196/16 = 12.25$

SUBCLASS MEANS.

$\bar{Y}_{11\cdot} = 84.49/8 = 10.56$ $\bar{Y}_{12\cdot} = 59.43/8 = 7.43$

$\bar{Y}_{21\cdot} = 63.12/8 = 7.89$ $\bar{Y}_{22\cdot} = 58.7/8 = 7.34$

$\bar{Y}_{31\cdot} = 97.39/8 = 12.17$ $\bar{Y}_{32\cdot} = 98.61/8 = 12.33$

$Y_{\cdot\cdot}$

	Fac. A 1	2	$Y_{i\cdot}$	$\bar{Y}_{i\cdot}$	A_i
Fac. B. 1	10.56	7.43		→	
2	7.89	7.34		→	
3	12.17	12.33		→	
$Y_{\cdot j}$	⋮				
$\bar{Y}_{\cdot j}$					
$\hat{B}_j$					

(3) $SS(Total) = \sum Y_{ijk}^2 - Y_{...}^2/N = 5065.153 - (461.74)^2/48$

$= 623.41$

$SS(Treat.) = \sum \frac{Y_{ij.}^2}{n} - Y_{...}^2/N = (84.49^2 + 59.43^2 + 63.12^2$
$58.7^2 + 97.39^2 + 98.61^2)/8 - \frac{(461.74)^2}{48}$

$= 221.89$

$SS(A) = \sum \frac{Y_{.j.}^2}{bn} - Y_{...}^2/N = \frac{245^2 + 216.74^2}{24} - \frac{461.74^2}{48}$

$= 16.64$

$SS(B) = \sum \frac{Y_{i..}^2}{an} - Y_{...}^2/N = \frac{143.92^2 + 121.82^2 + 196^2}{16} - \frac{461.74^2}{48}$

$= 181.32$

$SS(AB) = SS(Treat) - SS(A) - SS(B)$

$= 221.89 - 16.64 - 181.32$

$= 23.93$

$SS(Residual) = SS(Tot.) - SS(Treat)$

$= 623.41 - 221.89$

$= 401.52$

Source	df	SS	MS(SS/df)	F(MS/EMS)	
Total	N-1=47	623.41			
Treat.	ab-1=5	221.89	44.38		
A(Species)	a-1=1	16.64	16.64	1.74	NS
B(Concentr.)	b-1 =2	181.32	90.66	9.48	S
AB	(a-1)(b-1)= 2	23.93	11.97	1.25	NS
Residual.	ab(n-1)=42	401.52	EMS=9.56		

(4) – Between species

$H_0: \mu_{As} = \mu_{Ad}$

$H_a: \mu_{As} \neq \mu_{Ad}$

$F = 1.74$

critical $F_{.05,1,42} = 4.08$

Since $1.74 < 4.08$, No difference between species

– Between seawater concentrations

$H_0: \mu_{50} = \mu_{75} = \mu_{100}$

H_a: At least one is different from the rest.

$F = 9.48$

$F_{.05,2,42} = 3.23$

Since $9.48 > 3.23$, Significant difference between concentrations

– $H_0: \mu_{As100} = \mu_{As75} = \mu_{As50} = \mu_{Ad100} = \mu_{Ad75} = \mu_{Ad50}$

H_a: At least one is different from the rest.

$F = 1.25$

$F_{.05,2,42} = 3.23$

Since $1.25 < 3.23$, No difference in species x concentrations.

(5) $H_0: \mu_{AS} = \mu_{Ad}$

$H_a: \mu_{AS} \neq \mu_{Ad}$ (two tailed)

$$t = \frac{\bar{Y}_{AS} - \bar{Y}_{Ad}}{\sqrt{EMS \times \frac{2}{n}}} = \frac{10.21 - 9.03}{\sqrt{9.56 \times \frac{2}{24}}} = 1.32$$

$t_{.05, 42} = 2.021$ Also $t^2 = 1.748$

Since $1.32 < 2.021$, No difference between species.

6)

	Source	df	L: 100%	75%	50%	$\frac{\Sigma c_i^2}{n_i}$	SS	MS	F
	Salinity	2					181.46	90.73	
100 vs. 50	Linear Salinity	1	$-1(9)$	$+0(7.62)$	$+1(12.25) = 3.25$	.25	84.76	84.76	8.5
75 vs. 100+50	Quad. Salinity	1	$1(9)$	$+ -2(7.62)$	$+1(12.25) = 6.01$	.75	95.56	95.76	9.6

Since $F_{.05, 1, 49} = 4.08$, there is a significant difference in 100 vs. 50 and 100 + 50 vs. 2(75).