Yalcin YAVAS

STAT 512
C. T. GASKINS

Problem

1. The following data on the density of small bricks resulted from an experiment involving 3 different sizes of powder particles, 3 pressures, and 3 temperatures of firing. The 27 combinations of the 3 x 3 x 3 factorial were run in duplicate.

5.0 → 4815
12.5 → 5344
20.0 → 5808

		(A) Temperature							
(B) Size	(C) Pressure	1900		2000		2100			
5-10	5.0	340	375 = 715	316	386 : 702	374	350 : 724	2141	6411
	12.5	388	370 = 758	338	214 : 552	334	366 : 700	2010	
	20.0	378	378 : 756	348	378 : 726	380	398 : 778	2260	
10-15	5.0	260	244 : 504	388	304 : 692	266	234 : 500	1696	5460
	12.5	322	342 : 664	300	420 : 720	234	258 : 492	1876	
	20.0	330	298 : 628	260	366 : 626	350	284 : 634	1888	
15-20	5.0	134	140 : 274	146	194 : 340	152	212 : 364	978	4096
	12.5	186	30 : 216	412	428 : 840	194	208 : 402	1458	
	20.0	40	210 : 250	436	490 : 926	230	254 : 484	1660	
			4765		6124		5078	15967	

N = 54

a. Show the linear model for the experiment and define the components of it.

b. Define the null hypotheses to be tested in the ANOVA table.

c. Compute the ANOVA table.

d. Indicate which factors and interactions are significant.

e. Set up tables of means for main effects and interaction.

f. Summarize the results of the experiment in one or two short paragraphs.

BRICKS.512

Yalcin YAVAS

a) $Y_{ijkl} = \mu + A_i + B_j + C_k + AB_{ij} + AC_{ik} + BC_{jk} + ABC_{ijk} + e_{ijkl}$

μ = population mean
A_i = main effect of temp. (A)
B_j = " " size (B)
C_k = " " pressure (C)

AB_{ij} = effect of temp x size interaction
AC_{ik} = " " temp x pressure "
BC_{jk} = " " size x press. "
ABC_{ijk} = " " temp x size x press. "
e_{ijkl} = residual error.

b) H_0 : $A_1 = \dots A_a$
$B_1 = \dots B_b$
$C_1 = \dots C_c$
$AB_{11} = \dots AB_{ab}$
$AC_{11} = \dots AC_{ac}$
$BC_{11} = \dots BC_{bc}$
$ABC_{111} = \dots ABC_{abc}$

c)

Source	df	SS	MS	F	$F_{c1\%}$
Total	53	525388.65			
Treat.	26	461339.15	17743.8	7.5	* 1.93
Temp (A)	2	56277.1	28138.6	11.9	* 3.35
Size (B)	2	150447.1	75223.4	31.7	* 3.35
press. (C)	2	27429.4	13714.7	5.78	* 3.35
AxB	4	127467	31866.8	13.43	* 2.73
AxC	4	11093.3	2773.3	1.17	2.73
BxC	4	22546	5636.5	2.38	2.73
AxBxC	8	66079.3	8259.9	3.48	* 2.31
Residual	27	64049.5	2372.2		

d) Significant difference between treatments, temp, size, press., temp x size, and temp x size x press. interactions.

e)

Size (B_j)

Temp (A_i)	5-10	10-15	15-20	$\bar{Y}_{i.}$	$Y_{i.}$	$\hat{A}_i$
1900	2229	1796	1740		5765	-61.93
	$\bar{x}$ = 743 (92.6)	599 (53.93)	247 (-146.52)	529.4		
2000	1980	2038	2106		6124	89.07
	$\bar{x}$ = 660 (-141.4)	679 (-16.4)	706 (157.81)	680.4		
2100	2202	1626	1250		5078	-27.15
	$\bar{x}$ = 734 (48.8)	542 (-37.5)	417 (-111.13)	564.22		
$\bar{Y}_{.j}$	712.33	606.6	455.11			
$Y_{.j}$	6411	5460	4096	$\bar{Y}_{..}$ = 591.37		
$\hat{B}_j$	120.96	15.3	-136.26	$Y_{..}$ = 15967		

$$\hat{AB}_{ij} = \bar{Y}_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..}$$

Press. (C_k)

Temp (A_i)	5.0	12.5	20.0	$\bar{Y}_{i\cdot\cdot}$	$Y_{i\cdot\cdot}$	$\hat{A}_i$
1900	1493 $\bar{x}$ = 497.6 (24.59)	1638 546 (14.15)	1634 544.6 (-38.75)	529.4	4765	-61.93
2000	1734 $\bar{x}$ = 578 (-46.07)	2112 704 (21.15)	2278 759.3 (24.93)	680.4	6125	89.07
2100	1588 $\bar{x}$ = 529.3 (21.48)	1594 531.3 (-35.3)	1896 632 (13.81)	564.22	5078	-27.15
$\bar{Y}_{\cdot\cdot k}$	535	593.7	645.3	591.37		
$Y_{\cdot\cdot k}$	4815	5344	5808			
$\hat{C}_k$	-56.37	2.41	53.96			

$$\hat{AC}_{ik} = \bar{Y}_{ij} - \bar{Y}_{i\cdot\cdot} - \bar{Y}_{\cdot\cdot k} + \bar{Y}_{\cdots}$$

Press (C_k)

Size (B_j)	5.0	12.5	20.0	$\bar{Y}_{ij}$	Y_{ij}	$\hat{B}_j$
5-10	2141 $\bar{x}$ = 713.6 (57.7)	2010 670 (-44.74)	2260 753.3 (-12.9)	712.3	6411	120.96
10-15	1696 $\bar{x}$ = 563.3 (15.04)	1876 625.3 (16.26)	1888 629.3 (-31.3)	606.6	5460	15.30
15-20	978 326 (-72.74)	1458 486 (28.48)	1660 553.3 (44.26)	455.1	4096	-136.26
$\bar{Y}_{\cdot\cdot k}$	535	593.7	649.3	591.37		
$Y_{\cdot\cdot k}$	4815	5344	5808			
$\hat{C}_k$	-56.37	2.41	53.96			

$$\hat{BC}_{jk} = \bar{Y}_{jk\cdot} - \bar{Y}_{\cdot\cdot k} - \bar{Y}_{ij\cdot} + \bar{Y}_{\cdots}$$

Orthogonal Polinomial contrast

Source	df	1900	2000	2100	$\Sigma c_i^2/n_i$	SS	MS	F
Temp.	2							
Linear	1	(-1)4765	+ 0(6124)	+ (1)5078	$-313^2/2(18)$	2721.4	2721.4	1.15
Quadl.	1	(1)4765	+ (-2)6124	+ (1)5078	$-2405^2/6(18)$	53555.8	53555.8	22.6 *
Residual	27					2372.2		

	5-10	10-15	15-20				
Size	2						
Linear	1	(-1)6411	+ (0)5460	+ (1)4096	$= 2315^2/2(18)$	148867	148867	*62.8
Quadl.	1	(1)6411	+ (-2)5460	+ (1)4096	$= -413^2/6(18)$	1579.7	1579.3	.67
Residual	27							

$F_{crit} = F_{.05,1,27} = 4.21$

Temp.

	Linear			Quadl.		
	-1	0	1	1	-2	1
	1900	2000	2100	1900	2000	2100
Linear						
(-1) 5-10	1	0	-1	-1	2	-1
(0) 10-15	0	0	0	0	0	0
(1) 15-20	-1	0	1	1	-2	1
Size / Quadl.						
(1) 5-10	-1	0	1	1	-2	1
(-2) 10-15	2	0	-2	-2	4	-2
(1) 15-20	1	0	1	1	-2	1

Source	df	Q
Temp x Size	4	
Lin x Lin.	1	(1) 2229 + (0) 1980 + (-1) 2202 + (0) 1796 + (0) 2038 + (0) 1626 + (-1) 740 + (0) 2106 + (1) 1250 = 537
Lin x Quad	1	(-1) 2229 + (0) 1980 + (1) 2202 + (2) 1796 + (0) 2038 + (-2) 1626 + (1) 740 + (0) 2106 + (1) 1250 = 2303
Quad x Lin	1	(-1) 2229 + (2) 1980 + (-1) 2202 + (0) 1796 + (0) 2038 + (0) 1626 + (1) 740 + (-2) 2106 + (1) 1250 = -2693
Quad x Quad	1	(1) 2229 + (-2) 1980 + (1) 2202 + (-2) 1796 + (4) 2038 + (-2) 1626 + (1) 740 + (-2) 2106 + (1) 1250 = -443
Residual	27	

Temp x Size	$\xi(i^2, \lambda_i)$	SS	MS	F	
L x L	4(6)	12015.4	12015.4	5.07	*
L x Q	12(6)	73664.0	73664.0	31.05	*
Q x L	12(6)	100725.7	100725.7	42.46	*
Q x Q	36(6)	908.56	908.56	.38	
Residual			2372.2		

$$F_{.05,1,27} = 4.21$$

(f) Significant difference in brick density due to differences in temp, size, press, temp x size & temp x size x press. The effect of temp. is is a quadratic relationship. However, the effect of size is Linear x Linear & Linear x Quadratic. That makes significant interaction between temp. & size. Quadr. x Quadr. interaction has no contribution to overall temp. by size interaction.